A THEORY OF MASS TRANSFER IN BINARY STARS

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Cehula & Pejcha (2023, MNRAS, 524, 471–490)

Cehula & Pejcha (2024, in prep.)

Symbiotic stars, weird novae, and related embarrassing binaries
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European Research Council

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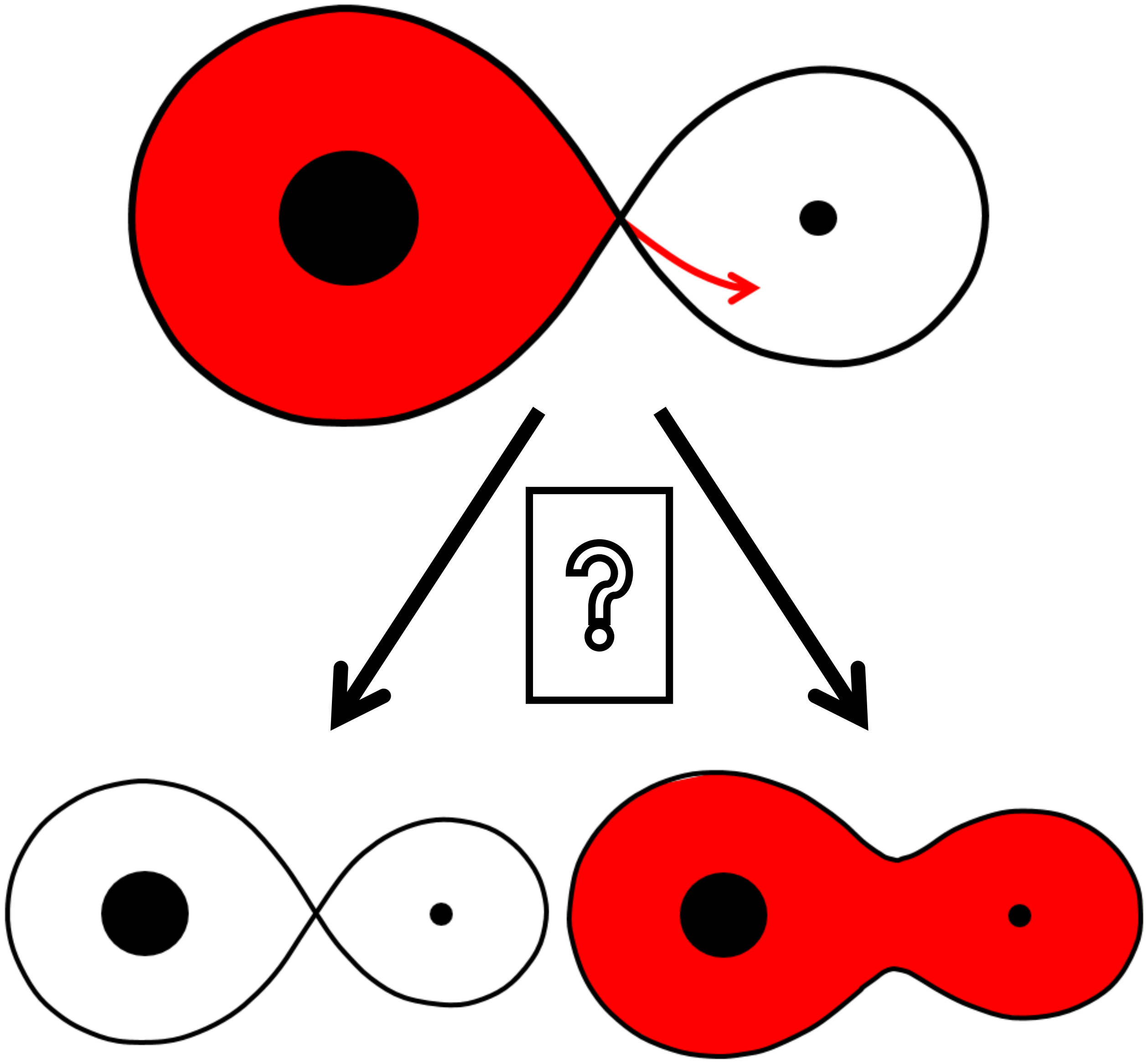
MOTIVATION

- mass transfer responsible for X-ray binaries, cataclysmic variables, type Ia supernovae, …
- understanding binary mass transfer => accurate differentiation between:
  1. stable mass transfer
  2. unstable mass transfer → common-envelope evolution
- standard mass-transfer models suffer from conceptual and practical difficulties => new model needed



MAIN GOAL

- donor's mass-loss rate:

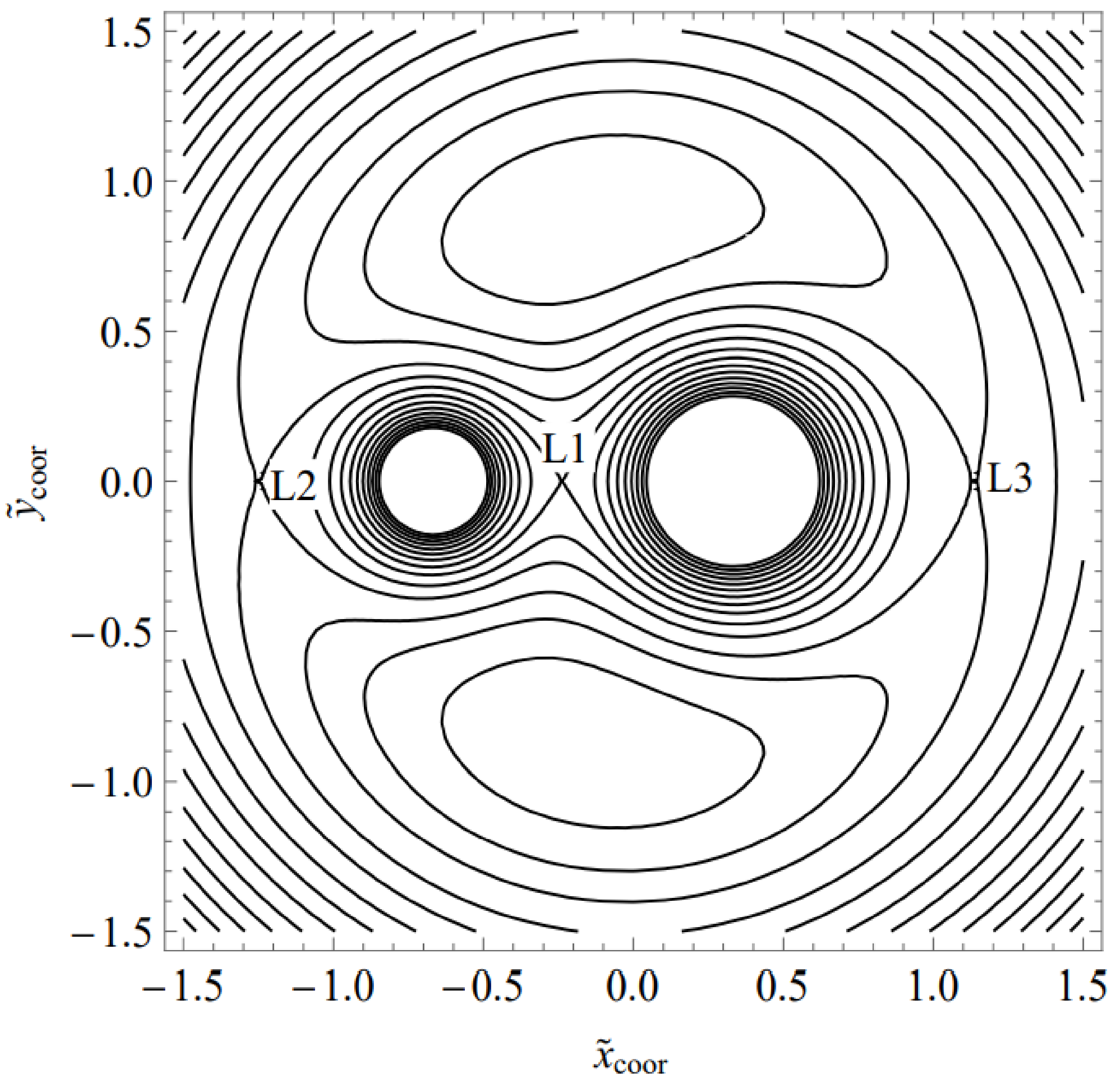
$$\dot{M}_{\rm d} = \dot{M}_{\rm d}(\delta R_{\rm d})$$

  - where $\delta R_{\rm d}$ is the relative radius excess:

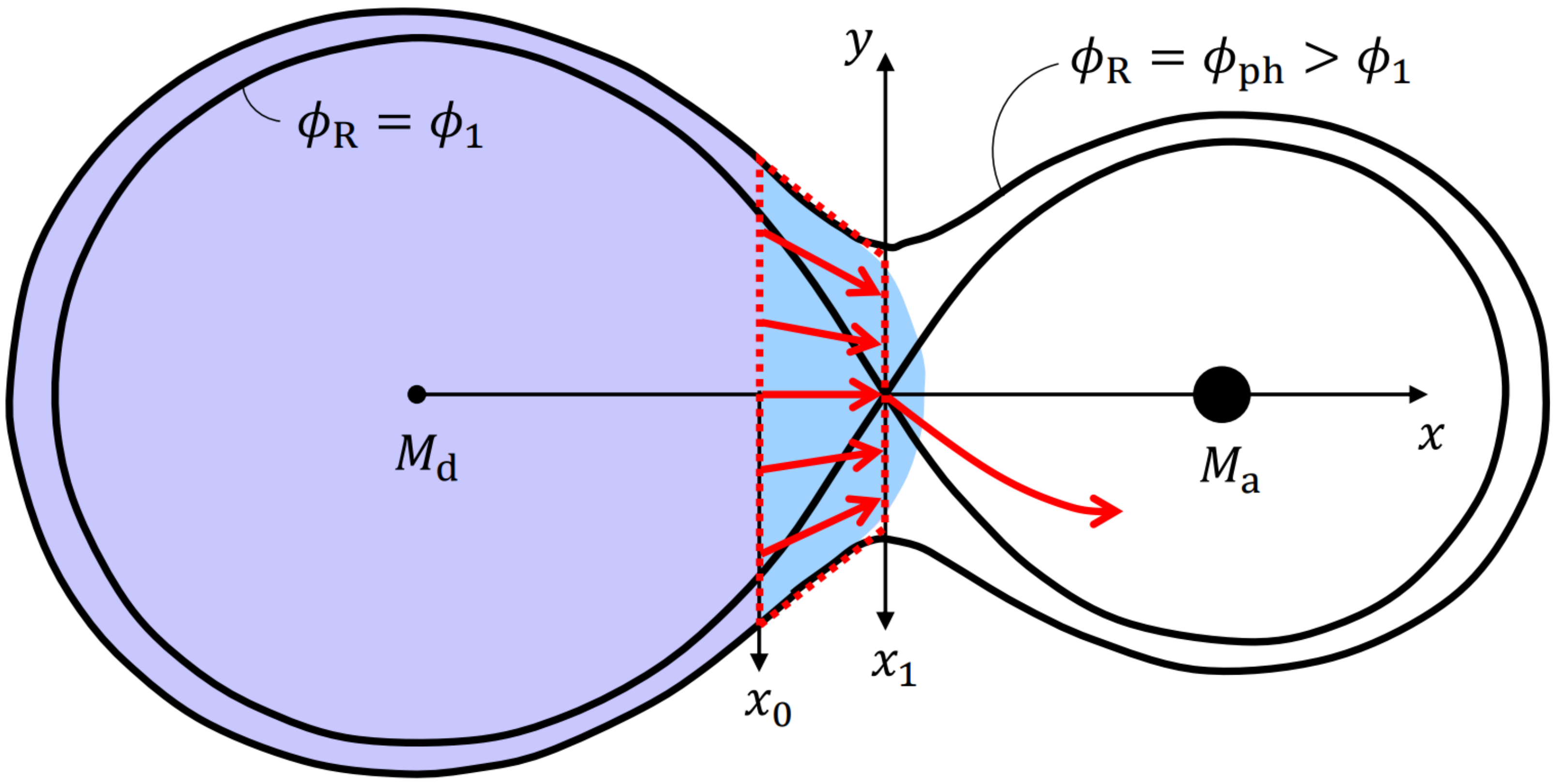
$$\delta R_{\rm d} \equiv \frac{\Delta R_{\rm d}}{R_{\rm L}} = \frac{R_{\rm d} - R_{\rm L}}{R_{\rm L}},$$

    $R_{\rm d}$ - donor's radius, $R_{\rm L}$ - Roche-lobe radius

  - serves as a **boundary condition** in a stellar evolution code (MESA)



NEW MODEL

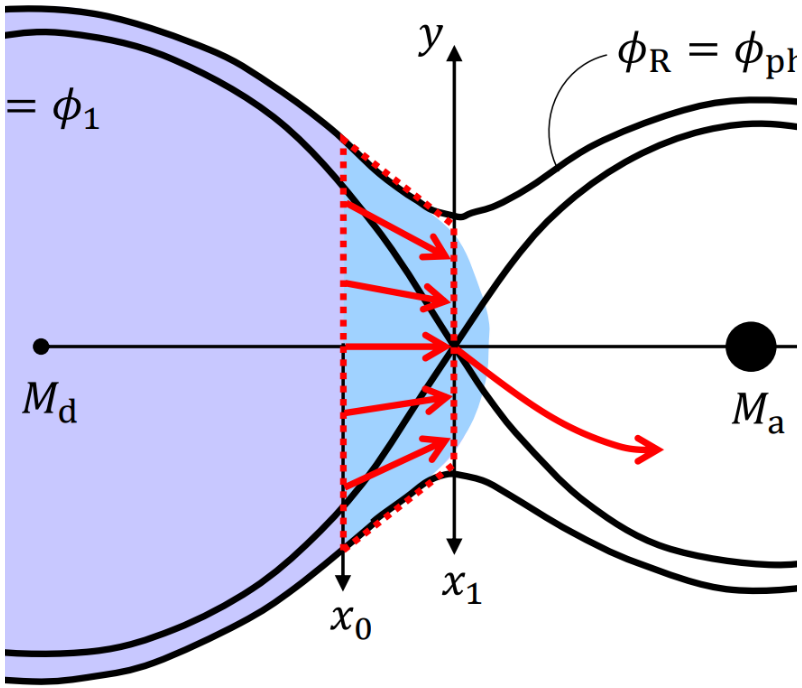


(Cehula & Pejcha 2023)

ADVANTAGES

- testing for systematic errors
- stellar interior (sonically connected) **influences** mass loss
- **possible** to include additional physics (radiation, mag. field, …)
- clear analogy with stellar winds – de Laval nozzle

NEW MODEL IN EQUATIONS



START

- 3D Euler equations with the Roche potential

ASSUMPTIONS

1. Stationarity: $\partial/\partial t \to 0$
2. Gas flow – effectively 1D ⇒ hydrostatic equilibrium in the perpendicular plane
3. Lowest order approximation of the Roche potential in the perpendicular plane
4. Polytropic approx. in the perpendicular plane

END

- 1D Euler equations with the Roche potential

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = 0,$$

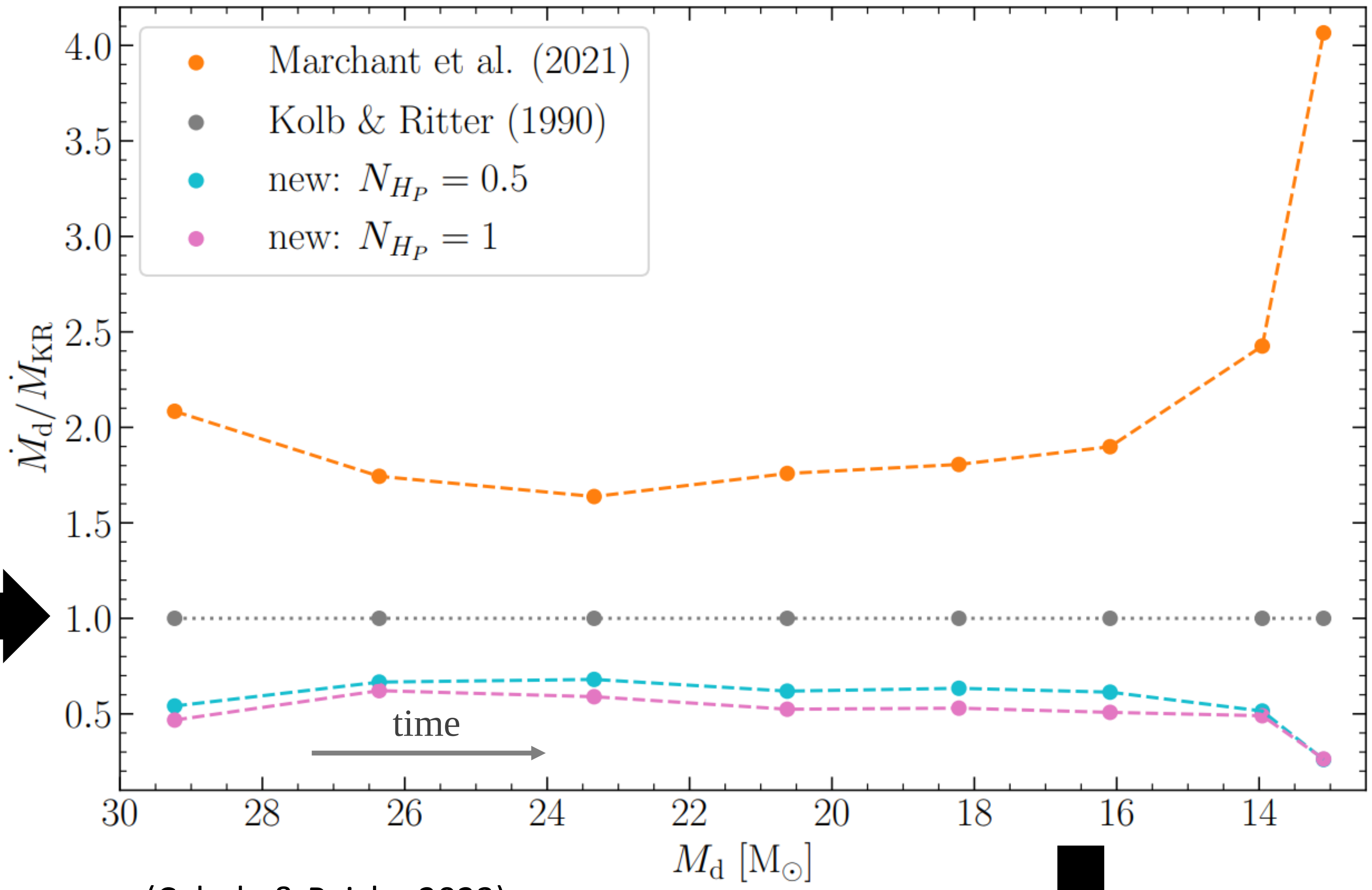
$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}(P Q_P) = -\frac{\mathrm{d}\phi_R}{\mathrm{d}x},$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\epsilon\frac{Q_P}{Q_\rho}\right) - \frac{P Q_P}{(\rho Q_\rho)^2}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = -\frac{\mathrm{d}}{\mathrm{d}x}\left(c_T^2\frac{Q_P}{Q_\rho}\right),$$

$$\dot{M}_{\mathrm{new}} = v\rho Q_\rho$$

RESULTS

- 30 $M_\odot$ star in a binary with 7.5 $M_\odot$ BH losing mass on thermal time scale evolved in Marchant et al. (2021) with MESA
- **a posteriori** $\dot{M}_{\rm d}$ comparison in different stages of star's evolution

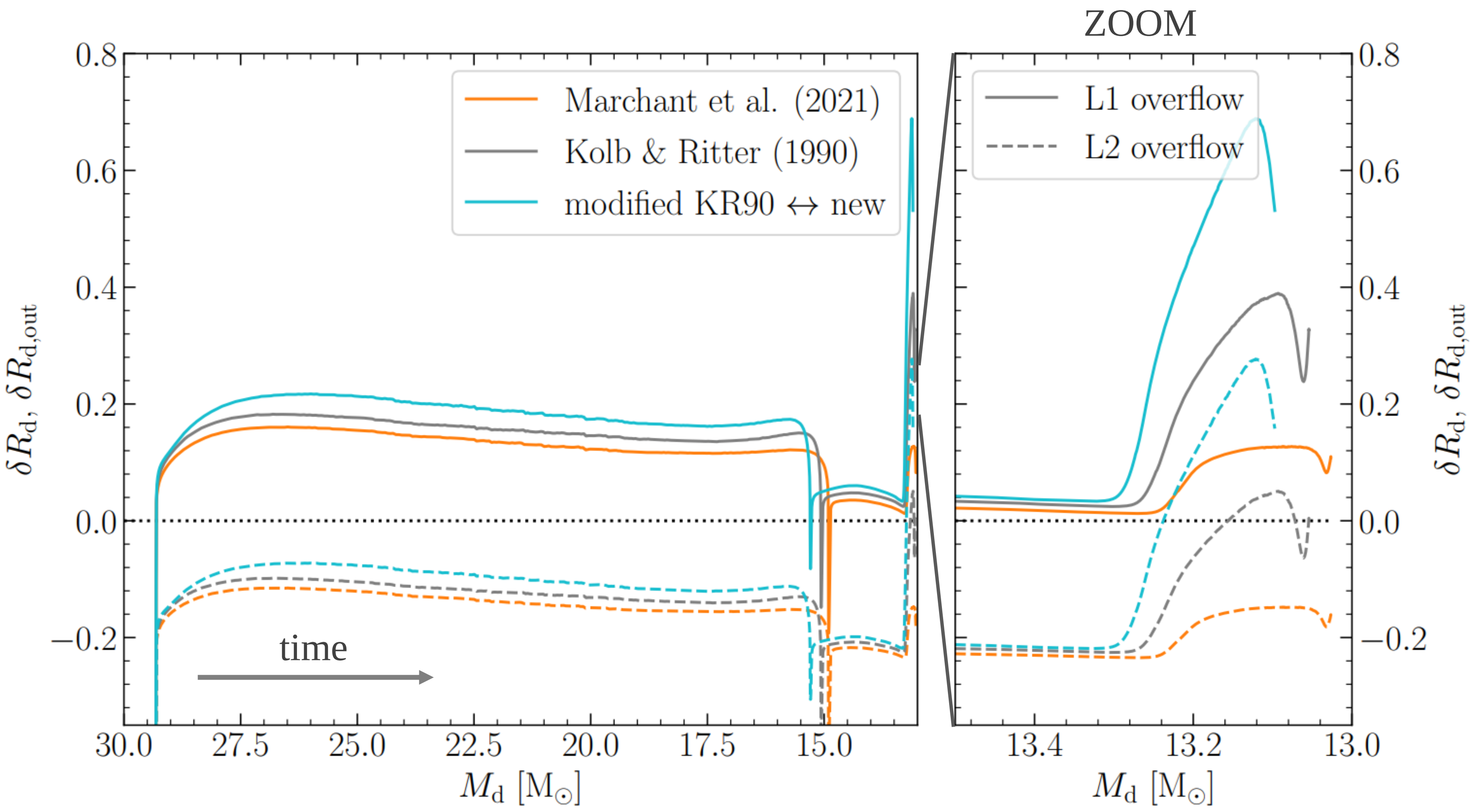


(Cehula & Pejcha 2023)

(MESA: Paxton et al. 2011, 2013, 2015, 2018, 2019)

RESULTS

- evolution **rerun** with 'KR90' mass-loss prescription decreased by a factor of 2 to simulate 'new' prescription ⇒ **less stable** mass transfer



(Cehula & Pejcha 2023)

CURRENT WORK

- implementation of radiative transfer

START

➢ 3D radiation hydrodynamics equations in the flux-limited diffusion approximation with the Roche potential

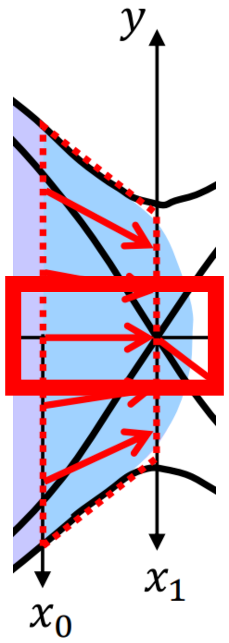
ASSUMPTIONS

1. Stationarity: $\partial/\partial t \to 0$
2. Gas flow – 1D: $\partial/\partial y \to 0$, $\partial/\partial z \to 0$
3. LTE: $aT^4 - E_{\mathrm{rad}} = 0$
4. Optically thick limit: $\lambda \to 1/3$

END

➢ 1D radiation hydrodynamics equations with the Roche potential and **radiative flux**

perpendicular plane



$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x} = 0,$$

$$\dot{m} = v\rho$$

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{gas}}}{\mathrm{d}x} = \frac{\kappa}{c}F_{\mathrm{rad}} - \frac{\mathrm{d}\phi_{\mathrm{R}}}{\mathrm{d}x},$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(\epsilon_{\mathrm{tot}}\rho + P\right)v + F_{\mathrm{rad}}\right] = 0,$$

$$F_{\mathrm{rad}} = -\frac{c}{\kappa}\frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}x},$$

$$P_{\mathrm{rad}} = \frac{1}{3}aT^4, \quad E_{\mathrm{rad}} = aT^4.$$

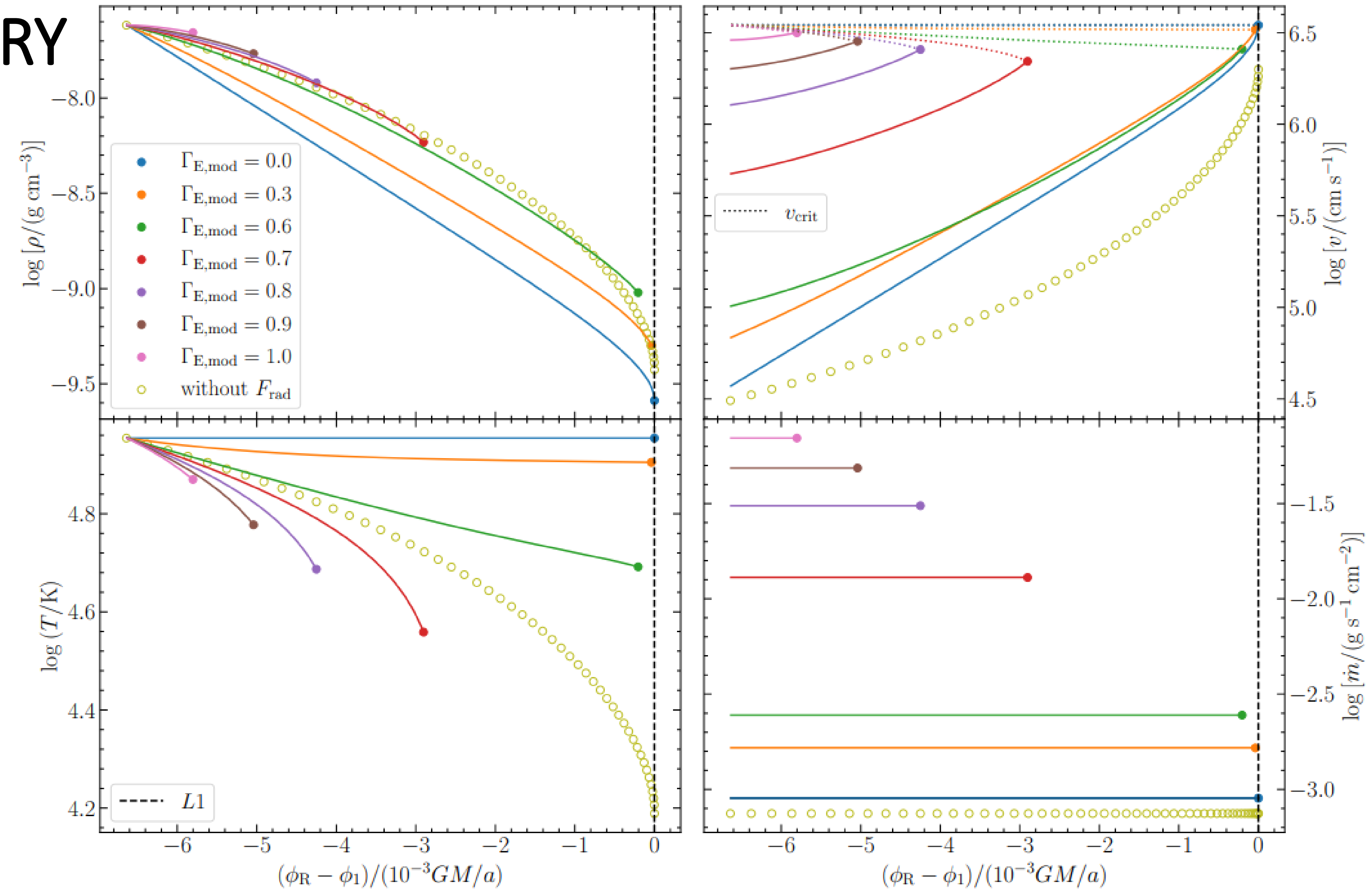
PRELIMINARY RESULTS

SETUP

- $M_\mathrm{d} = 30\mathrm{M}_\odot$
- $q = 1$
- $\delta R_\mathrm{d} = 0$
- $\kappa = 1.2\ \mathrm{cm}^2\ \mathrm{g}^{-1}$
- $P_\mathrm{gas} = \frac{k}{\mu m_u}\rho T$
- $\phi_\mathrm{R} = \zeta(x)\frac{GM_\mathrm{d}}{R_\mathrm{L}+x-x_1}$
- $\Gamma_\mathrm{E,mod}$ – modified Eddington factor

RESULTS

- shift of the critical point



(Cehula & Pejcha 2024, in prep.)

PRELIMINARY RESULTS

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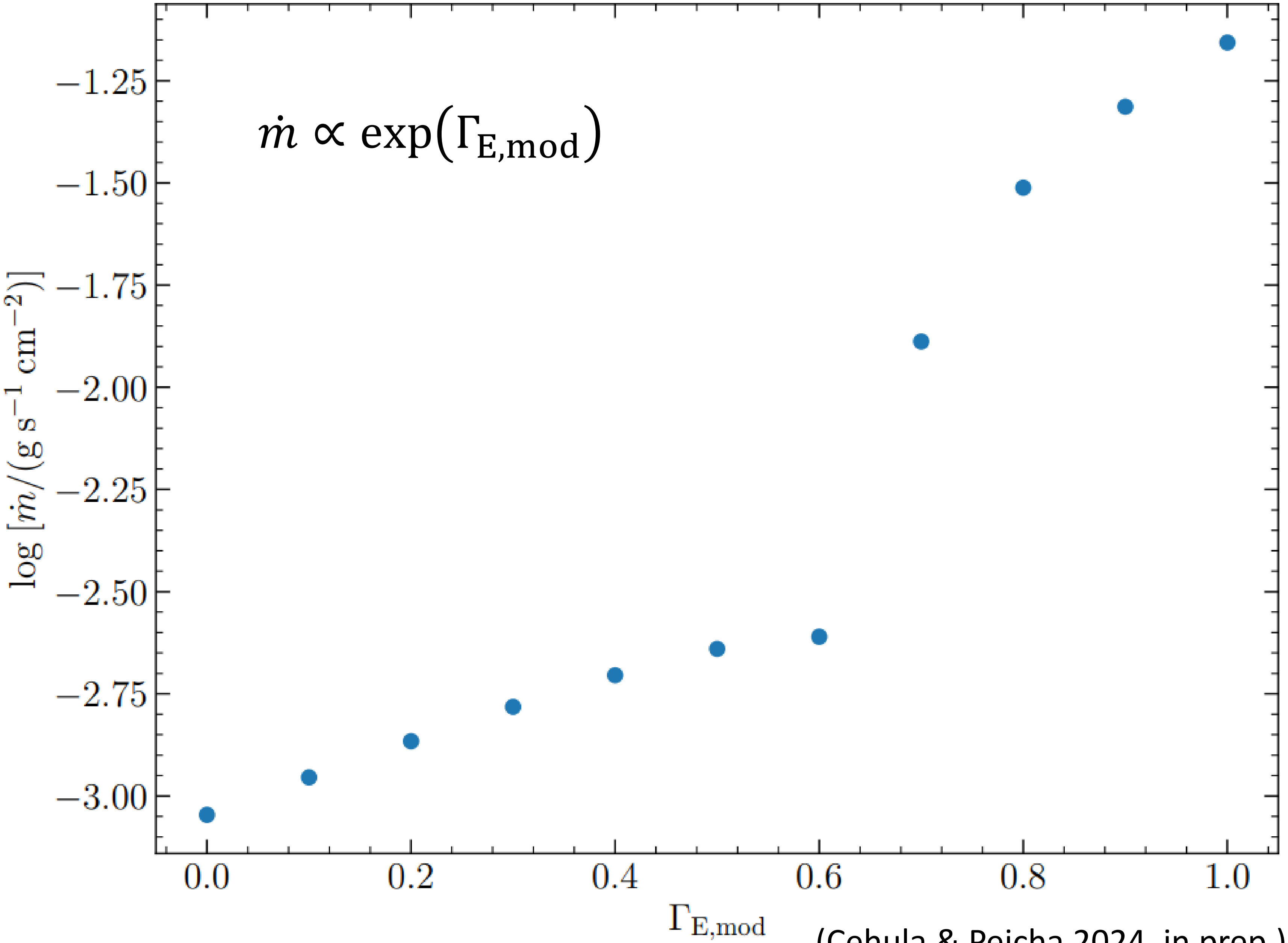
- $\Gamma_{\mathrm{E,mod}}$ – modified Eddington factor

RESULTS

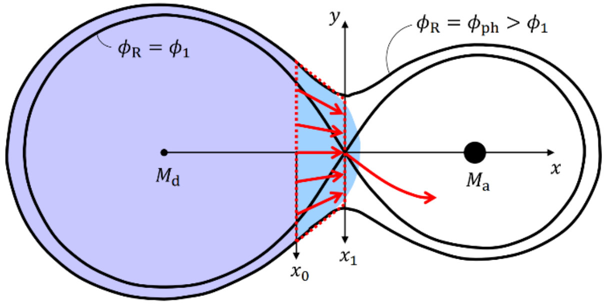
- $\dot{m} \propto \exp(\Gamma_{\mathrm{E,mod}})$



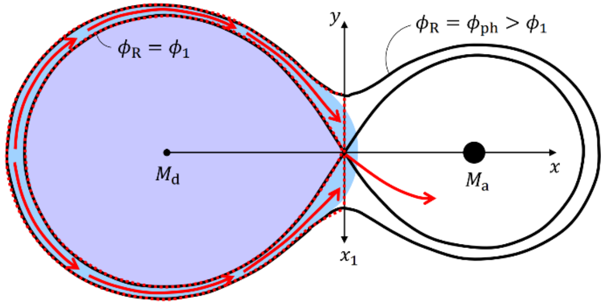
(Cehula & Pejcha 2024, in prep.)

SUMMARY

- comparison with Marchant et al. (2021):
  - factor of 4 lower $\dot{M}_{\mathrm{d}}$ ⇒ greater δR_{d} for given $\dot{M}_{\mathrm{d}}$ ⇒ less stable mass transfer ⇒ **favors CEE** over stable mass transfer
- comparison with Kolb & Ritter (1990):
  - factor of 2 difference in $\dot{M}_{\mathrm{d}}$
- testing for **systematic differences** between the two models
- current work:
  - including additional physics ≡ radiative transfer (**not** possible using the standard model)
  - preliminary results: $\dot{m} \propto \exp\left(\Gamma_{\mathrm{E,mod}}\right)$



vs.



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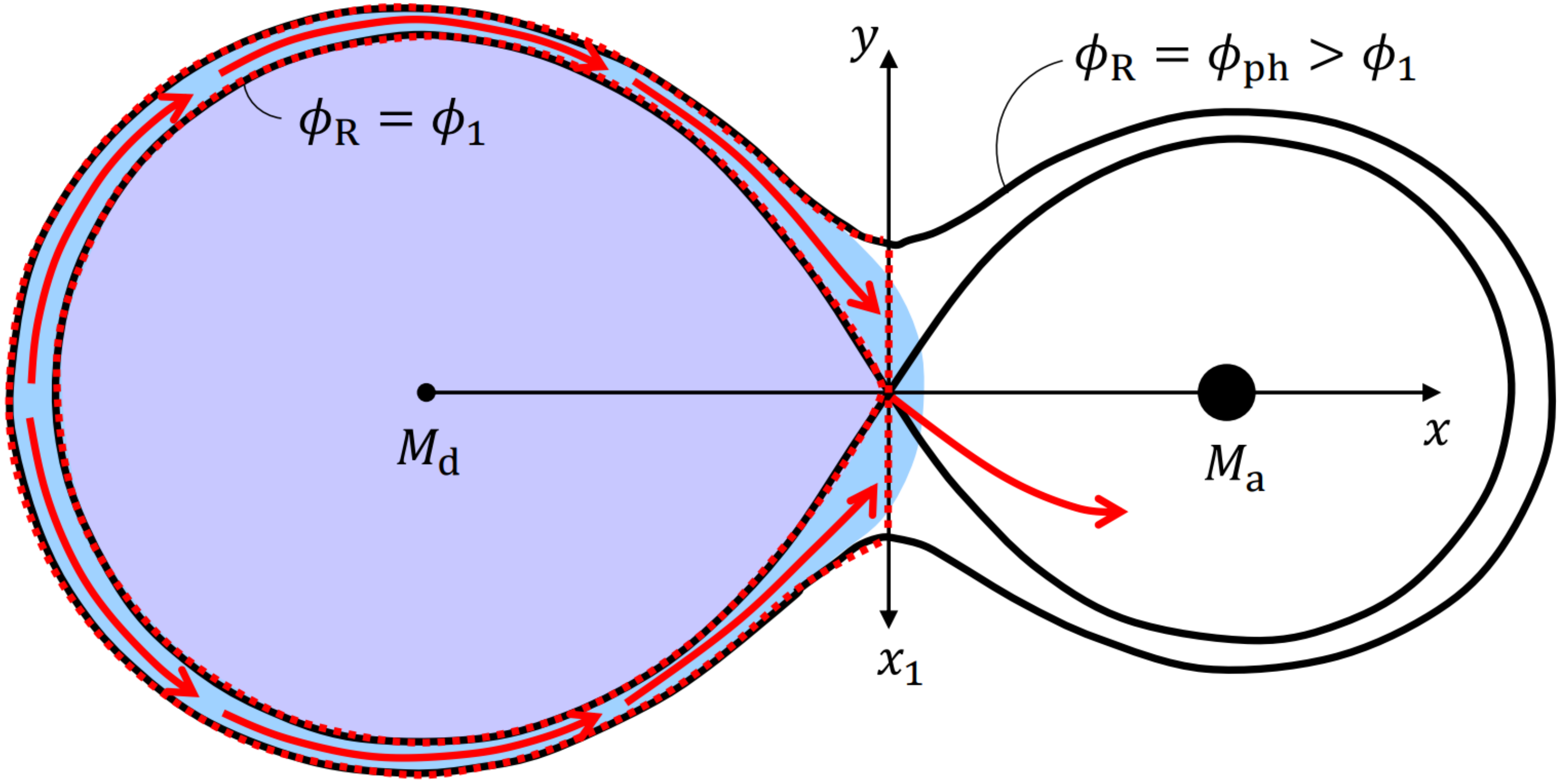
BACKUP SLIDES

STANDARD MODEL <–> POTENTIAL



- possible systematic errors
- **instant** optically thin → thick transition
- stellar interior (sonically connected) does **not** influence mass loss
- **not** possible to include additional physics (radiation, mag. field, …)

(Lubow & Shu 1975, Ritter 1988, Kolb & Ritter 1990, Pavlovskii & Ivanova 2015, Jackson et al. 2017, Marchant et al. 2021)

$$\dot{M}_{KR} = \left.\frac{\mathrm{d}Q}{\mathrm{d}\phi}\right|_{L1} \int_{\phi_1}^{\phi_{ph}} F_3(\Gamma) \left(\frac{k\bar{T}}{\bar{m}}\right)^{\frac{1}{2}} \bar{\rho}\mathrm{d}\bar{\phi},$$

(Kolb & Ritter 1990)

NEW MODEL IN EQUATIONS

START

- 3D Euler equations with the Roche potential:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + P\mathbf{I}) = -\rho \boldsymbol{\nabla} \phi_{\mathrm{R}},$$

$$\frac{\partial (\rho \epsilon_{\mathrm{tot}})}{\partial t} + \boldsymbol{\nabla} \cdot [(\rho \epsilon_{\mathrm{tot}} + P) \boldsymbol{v}] = 0,$$

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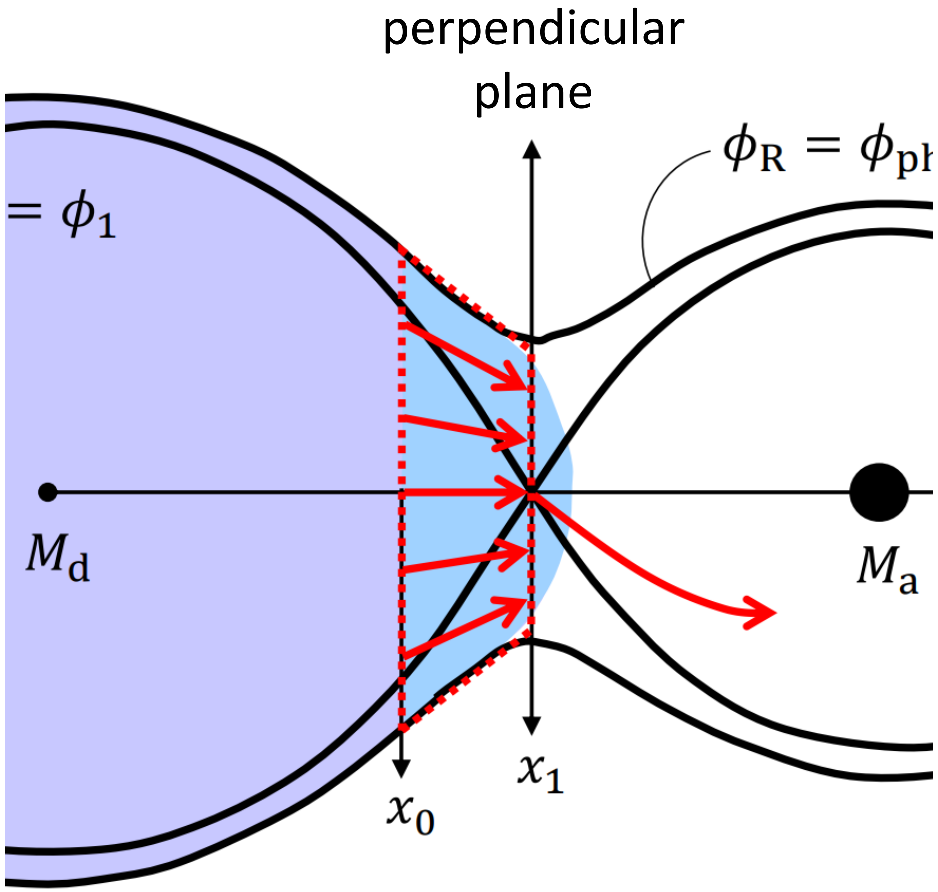

ASSUMPTIONS:

1. Stationarity
2. Gas flow – effectively 1D ⇒ hydrostatic equilibrium in the perpendicular plane
3. Lowest order approximation of the Roche potential in the perpendicular plane
4. Polytropic approx. in the perpendicular plane

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$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = 0,$$

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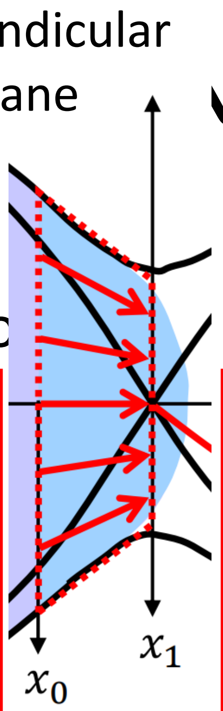
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\epsilon \frac{Q_P}{Q_\rho}\right) - \frac{P Q_P}{(\rho Q_\rho)^2}\frac{\mathrm{d}}{\mathrm{d}x}(\rho Q_\rho) = -\frac{\mathrm{d}}{\mathrm{d}x}\left(c_T^2 \frac{Q_P}{Q_\rho}\right),$$

where we are averaging in the perpendicular plane: $\rho Q_\rho = \int_Q \rho' \mathrm{dQ}, \quad P Q_P = \int_Q P' \mathrm{dQ},$

$\frac{Q_P}{Q_\rho} = \frac{\Gamma}{2\Gamma - 1}$, and: $\dot{M}_{\mathrm{new}} = v \rho Q_\rho$

NEW MODEL IN EC NS



START

- 3D Euler equations with the Roche po

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$
$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + P\mathbf{I}) = -\rho \nabla \phi_{\mathrm{R}},$$
$$\frac{\partial (\rho \epsilon_{\mathrm{tot}})}{\partial t} + \nabla \cdot [(\rho \epsilon_{\mathrm{tot}} + P) \boldsymbol{v}] = 0,$$

ASSUMPTIONS: +

1. Stationarity
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$\frac{Q_P}{Q_\rho} = \frac{\Gamma}{2\Gamma - 1}$, and:

$$\dot{M}_{\mathrm{new}} = v \rho Q_\rho$$

SOLUTION OF NEW EQUATIONS

- 1D Euler equations with the Roche potential:

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}\left(\rho Q_\rho\right) = 0,$$

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho Q_\rho}\frac{\mathrm{d}}{\mathrm{d}x}\left(P Q_P\right) = -\frac{\mathrm{d}\phi_\mathrm{R}}{\mathrm{d}x},$$

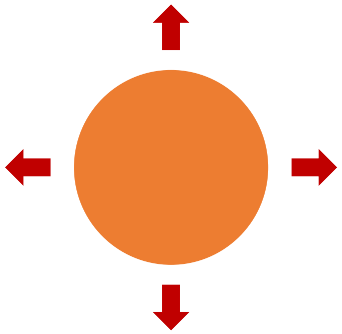
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\epsilon\frac{Q_P}{Q_\rho}\right) - \frac{P Q_P}{\left(\rho Q_\rho\right)^2}\frac{\mathrm{d}}{\mathrm{d}x}\left(\rho Q_\rho\right) = -\frac{\mathrm{d}}{\mathrm{d}x}\left(c_T^2\frac{Q_P}{Q_\rho}\right),$$

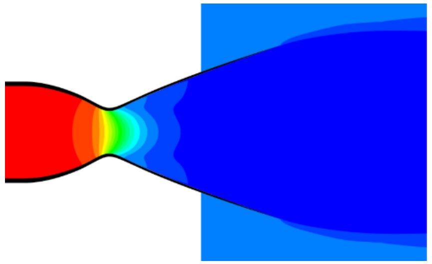
- 2-point BVP ⇒ numerical relaxation (Press et al. 2007)

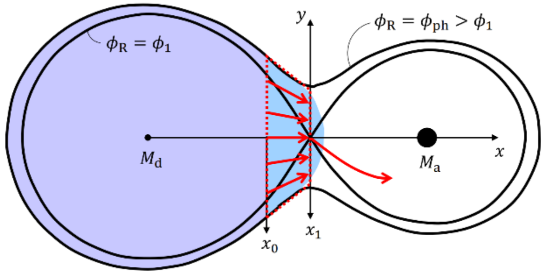
- we still need the **EQUATION OF STATE**
  - isothermal: $P = K\rho$
  - polytropic: $P = K\rho^\Gamma$
  - (isothermal and polytropic) } algebraic solution
  - realistic: MESA EOS module

(Saumon, Chabrier, & van Horn 1995; Irwin 2004; Timmes & Swesty 2000; Potekhin & Chabrier 2010; Jermyn et al. 2021)

STELLAR WINDS ≡ WAY TO NEW MT MODEL







- analogies between:
  - ➢ 1D isothermal stellar wind
  - ➢ flow through a rocket nozzle
  - ➢ **new model:** mass transfer through the nozzle created by the Roche potential around L1
- hydrodynamic equations governing 1D isothermal stellar wind:

$$-\dot{M}_* = 4\pi r^2 \rho(r)\, v(r) = \text{const},$$

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM_*}{r^2} = 0,$$

$$T(r) = T = \text{const}.$$

- assuming ideal gas EOS: $$\frac{1}{v}\frac{dv}{dr} = \frac{\frac{2c_T^2}{r} - \frac{GM_*}{r^2}}{v^2 - c_T^2}$$

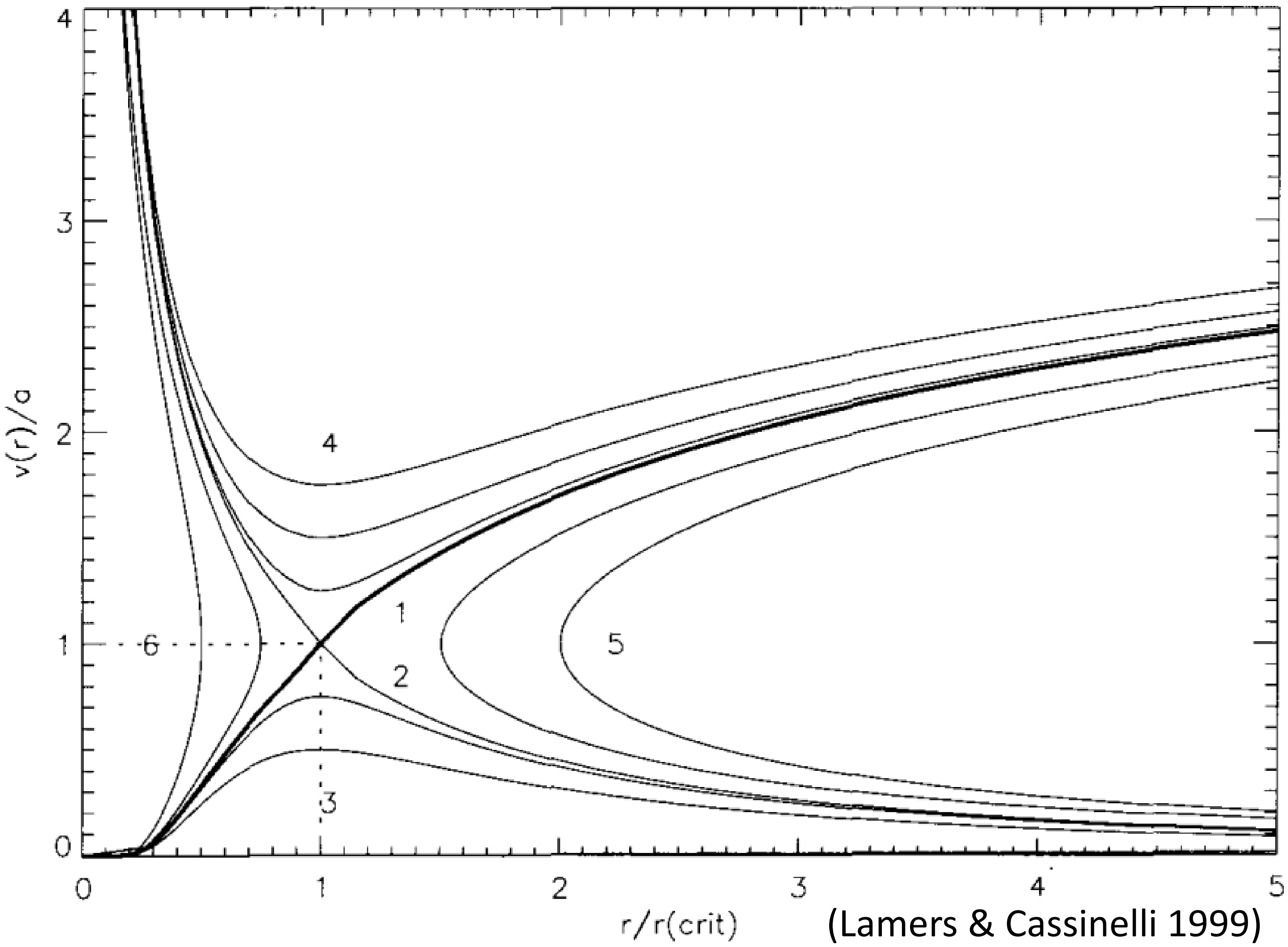
STELLAR WINDS ≡ WAY TO NEW MT MODEL

- solutions of:

$$\frac{1}{v}\frac{dv}{dr} = \frac{\frac{2c_T^2}{r} - \frac{GM_*}{r^2}}{v^2 - c_T^2}$$

- the critical point ($v = c_T$):

$$r_c = \frac{GM_*}{2c_T^2}$$



(Lamers & Cassinelli 1999)

ANALOGY TO ROCKET NOZZLES

- hydrodynamic equations governing isothermal gas flow through axially symmetric nozzle:

$$\dot{M}_N = \rho(l)v(l)A(l) = \rho_b v_b A_b = \text{const},$$

$$v\frac{dv}{dl} + \frac{1}{\rho}\frac{dP}{dl} = 0.$$

$$T(l) = T = \text{const}.$$

- assuming ideal gas EOS: $\frac{1}{v}\frac{dv}{dl} = \frac{\frac{c_T^2}{A}\frac{dA}{dl}}{v^2 - c_T^2},$

- the critical point ($v = c_T$): $dA/dl = 0$

ANALOGY TO ROCKET NOZZLES

- considering:

$$\frac{1}{v}\frac{dv}{dl} = \frac{\frac{c_T^2}{A}\frac{dA}{dl}}{v^2 - c_T^2},$$

- where ($A = \pi r_N^2$):

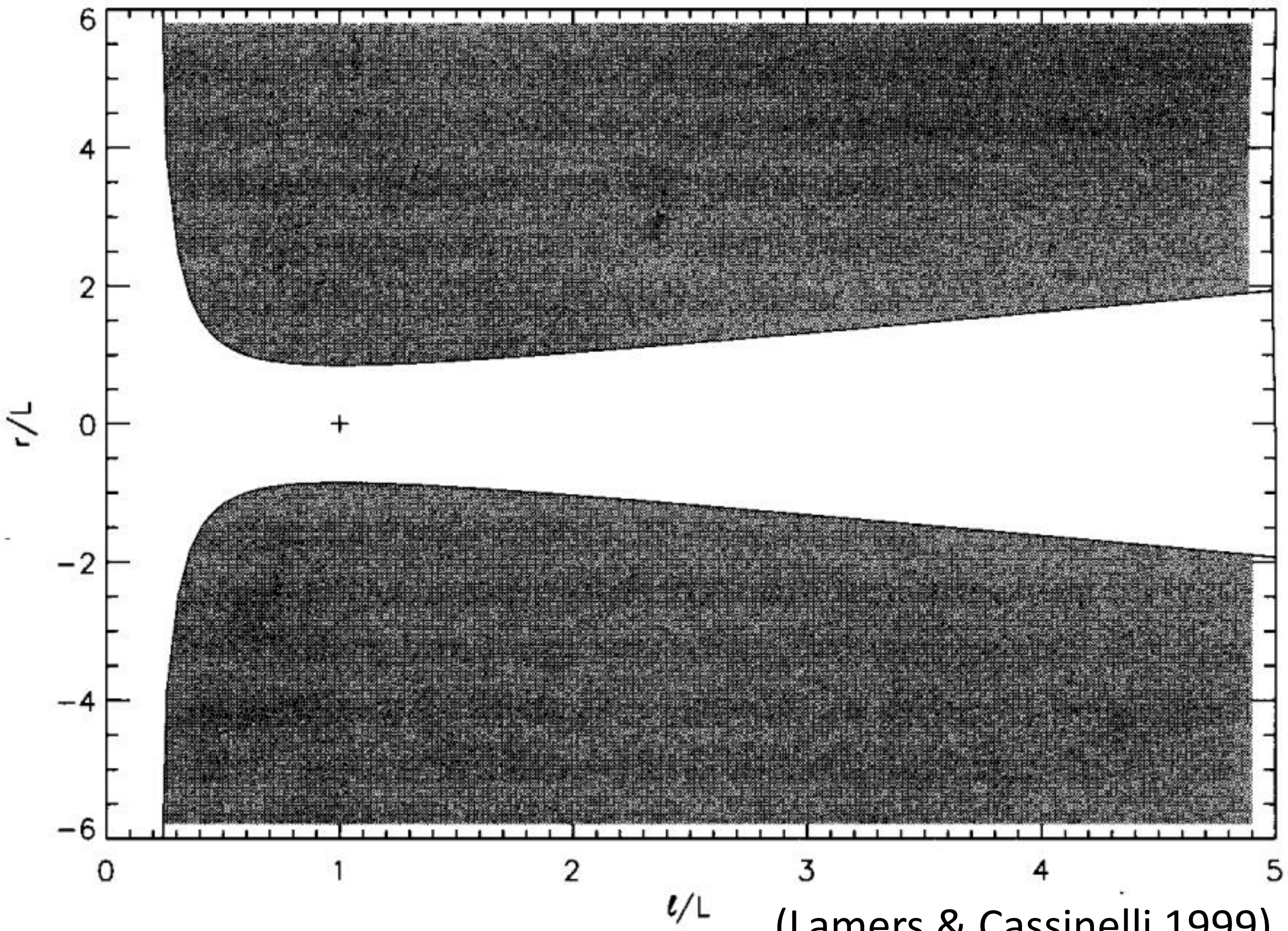
$$r_N(l) = \frac{l}{\pi}\exp\left(\frac{L}{l}\right), \quad \text{with} \quad L = \frac{GM_*}{2c_T^2},$$

- yields:

$$\frac{c_T^2}{A}\frac{dA}{dl} \equiv \frac{2c_T^2}{r} - \frac{GM_*}{r^2}, \quad \text{for} \quad l = r,$$

- i.e. the same momentum equation and velocity distribution as **isothermal wind**:

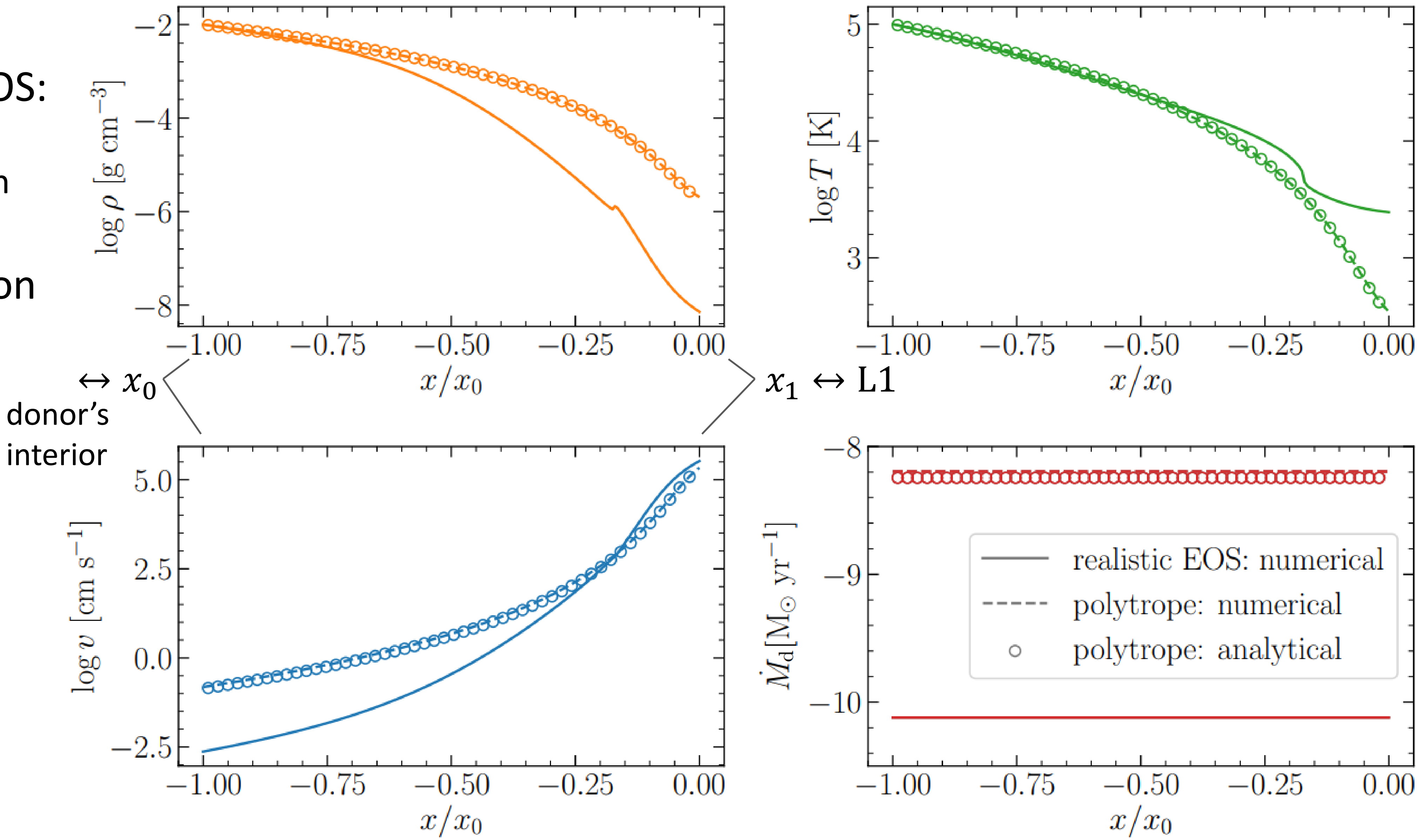
$$\frac{1}{v}\frac{dv}{dr} = \frac{\frac{2c_T^2}{r} - \frac{GM_*}{r^2}}{v^2 - c_T^2}$$



(Lamers & Cassinelli 1999)

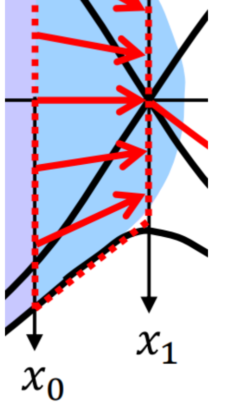
RESULTS

- polytropic vs. more realistic EOS:
  - ➢ factor of 10^2 difference in an extreme case!
- analytical solution agrees with the numerical for polytrope
- $\dot{M}_{\rm d}(x) = \text{const.}$



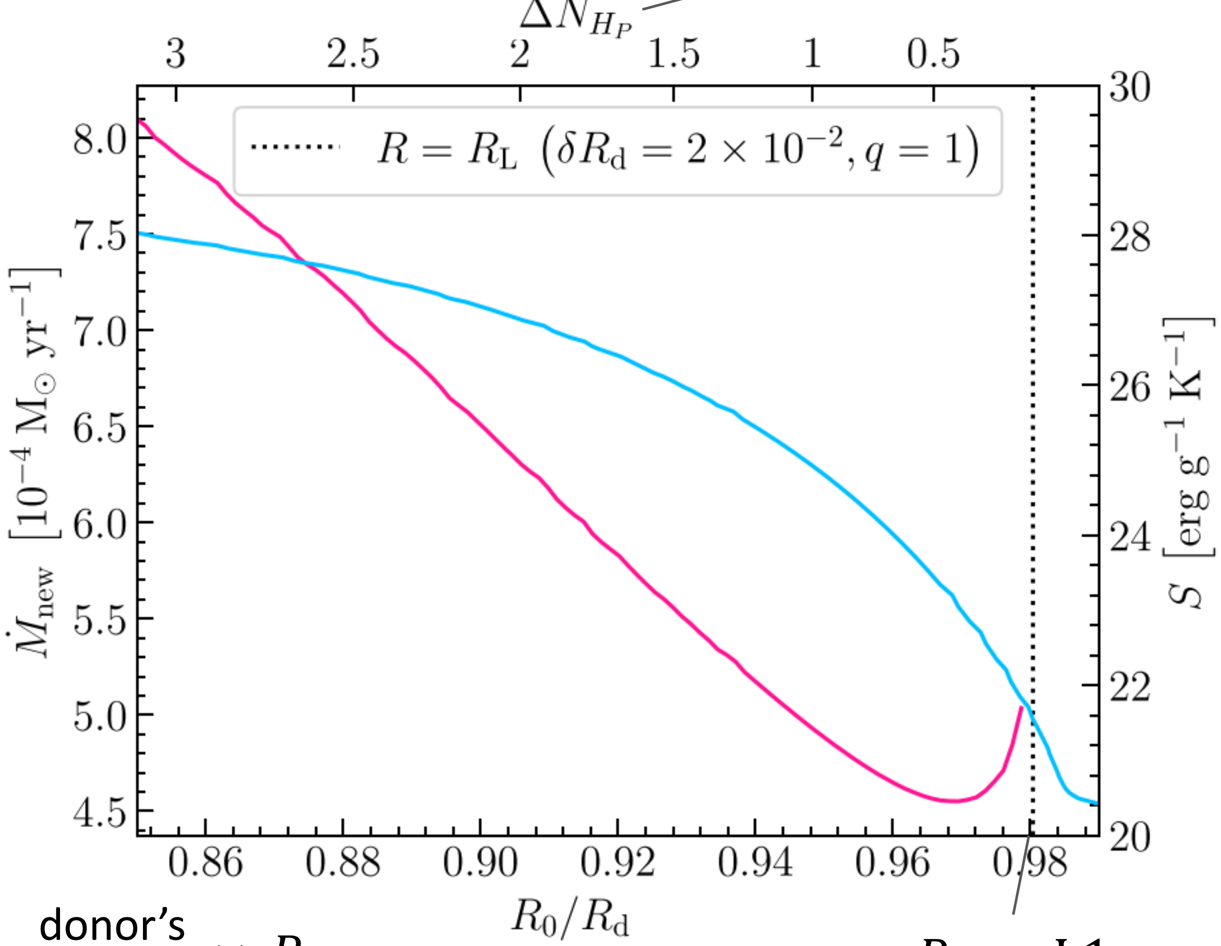
RESULTS

- $\dot{M}_{\rm new}(\Delta N_{H_P})$ <–> $\dot{M}_{\rm new}(x_0)$, $\delta R_{\rm d}$ = const.!



1M$_\odot$ donor on RGB

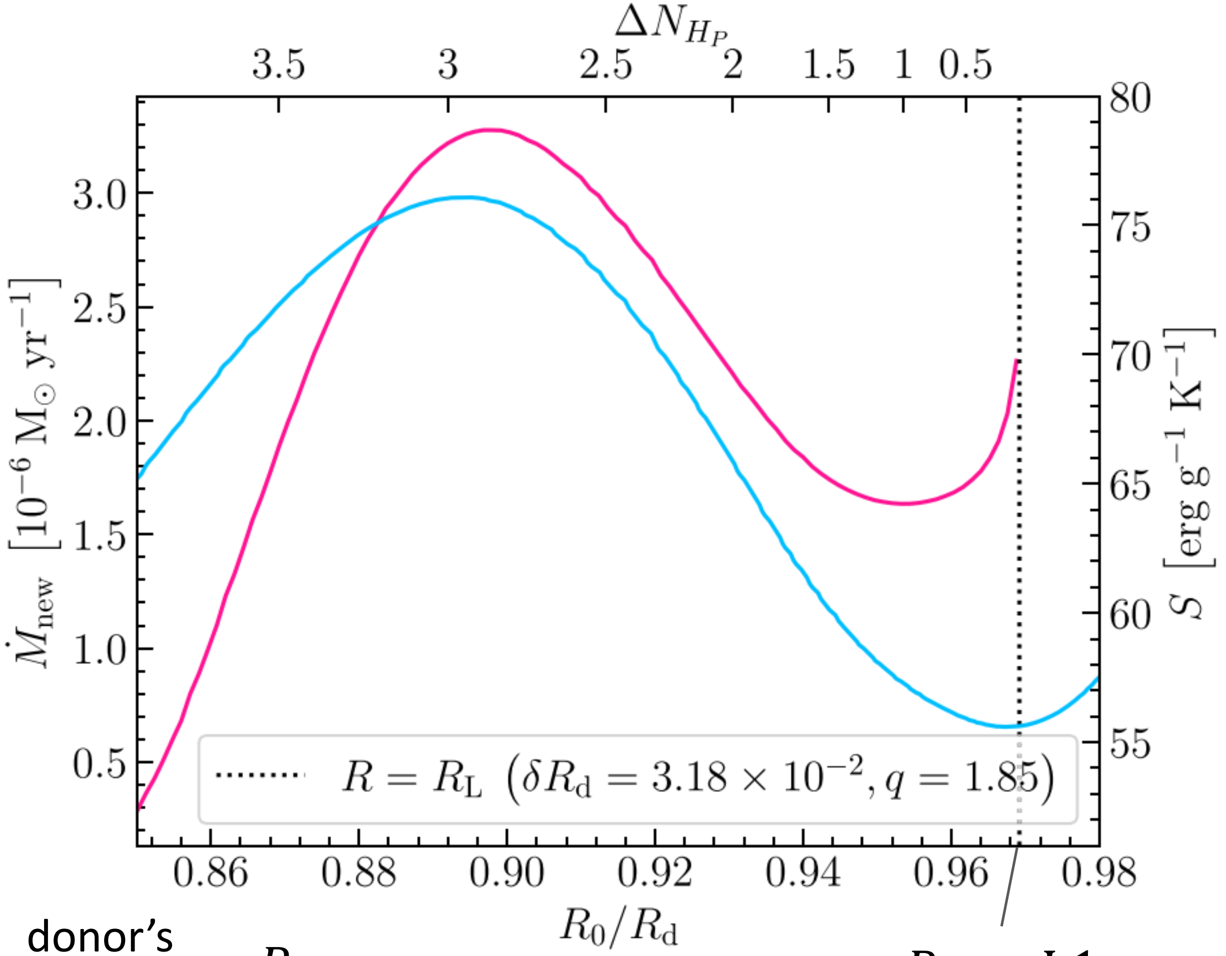
# of pressure scale heights



donor's interior $\leftrightarrow R_0$

$R_{\rm L} \leftrightarrow$ L1

30M$_\odot$ low-metallicity donor undergoing thermal MT



donor's interior $\leftrightarrow R_0$

$R_{\rm L} \leftrightarrow$ L1

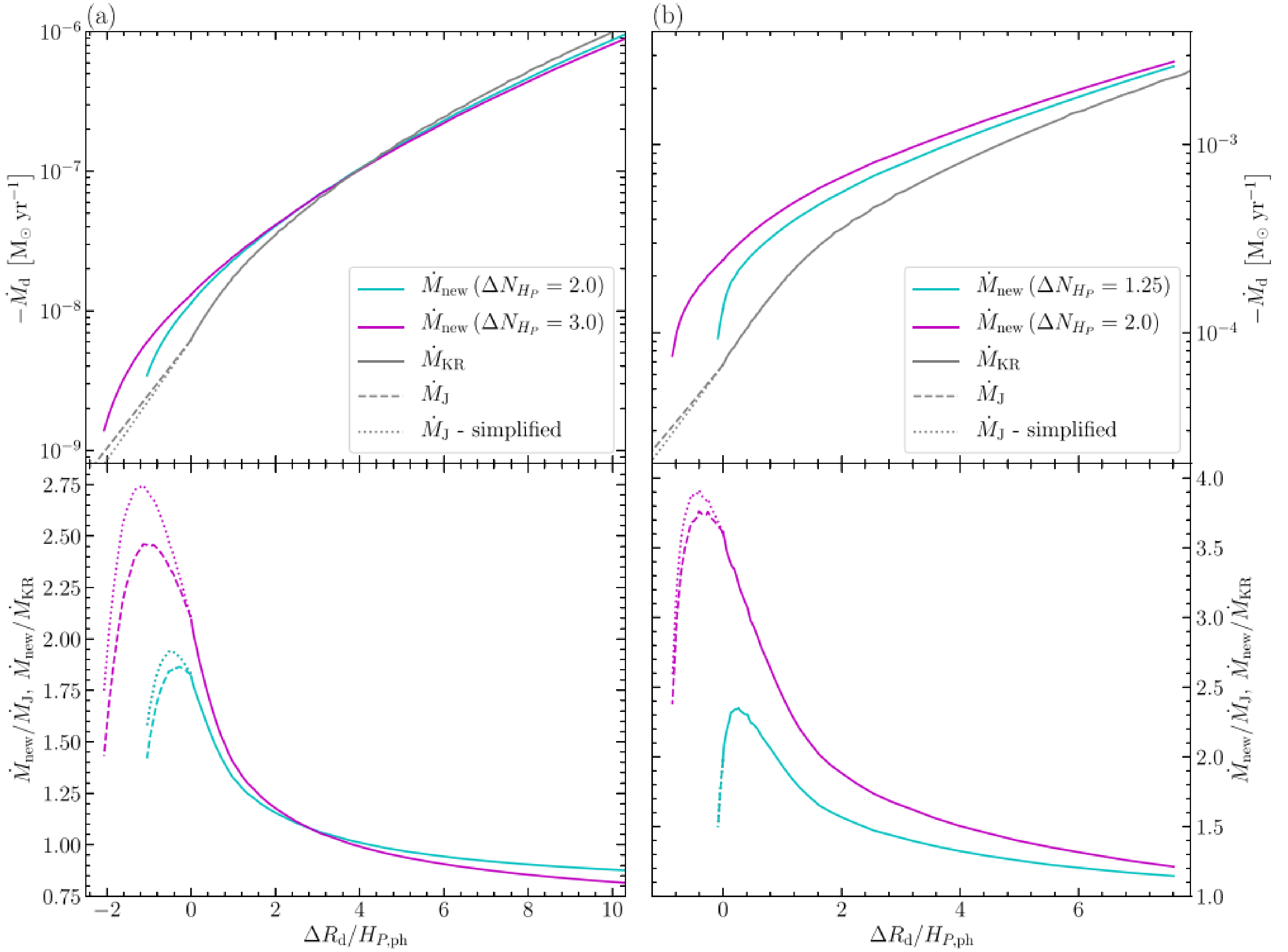
RESULTS

MT rate comparison

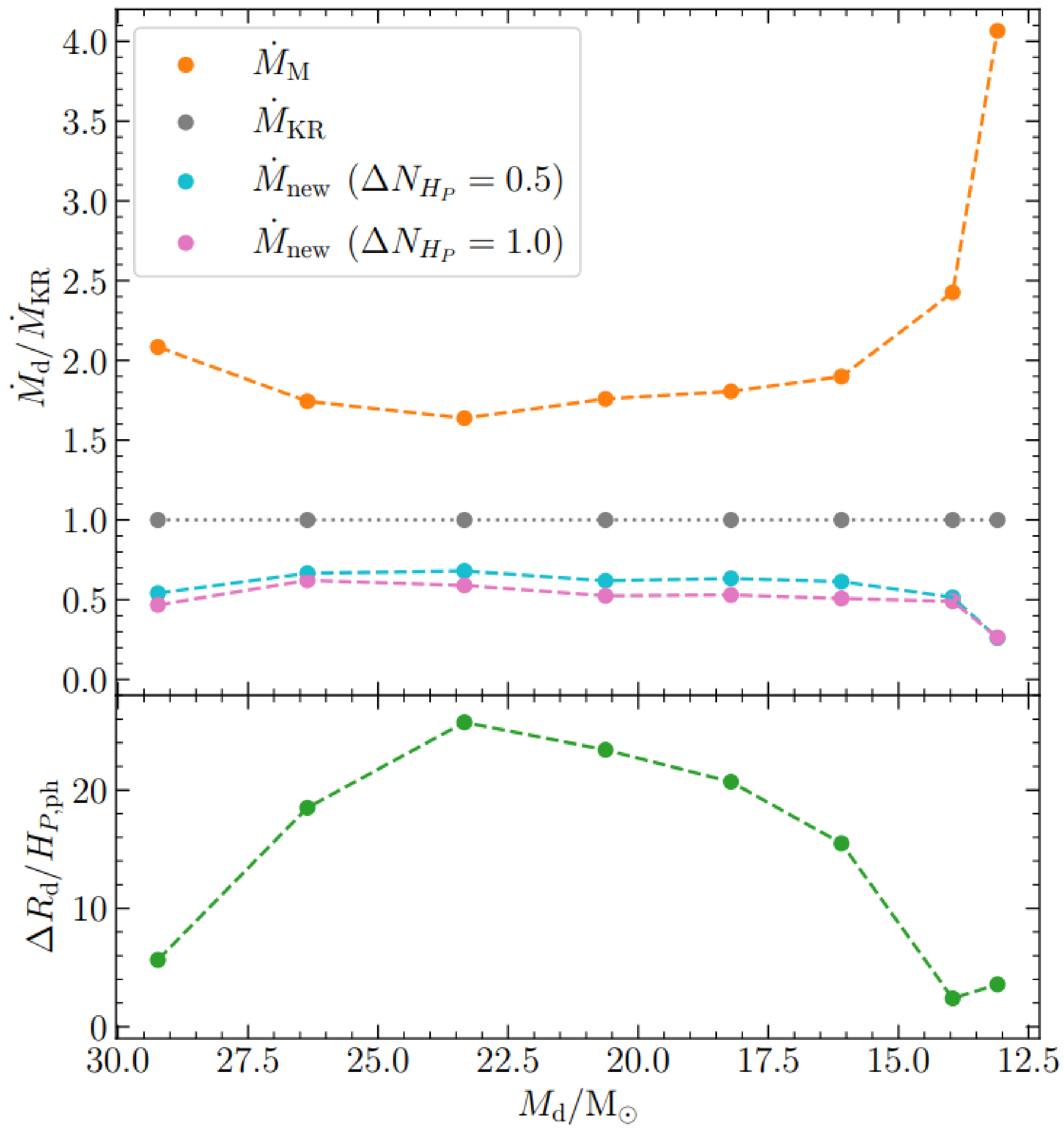
- $\dot{M}_{\mathrm{new}}(\Delta R_{\mathrm{d}})$, $\Delta N_{H_P} = \text{const.!}$
- vs. optically thin (Jackson et al. 2017)
- vs. optically thick (Kolb & Ritter 1990)

(a) $1\mathrm{M}_\odot$ donor on the main sequence

(b) $1\mathrm{M}_\odot$ donor on RGB

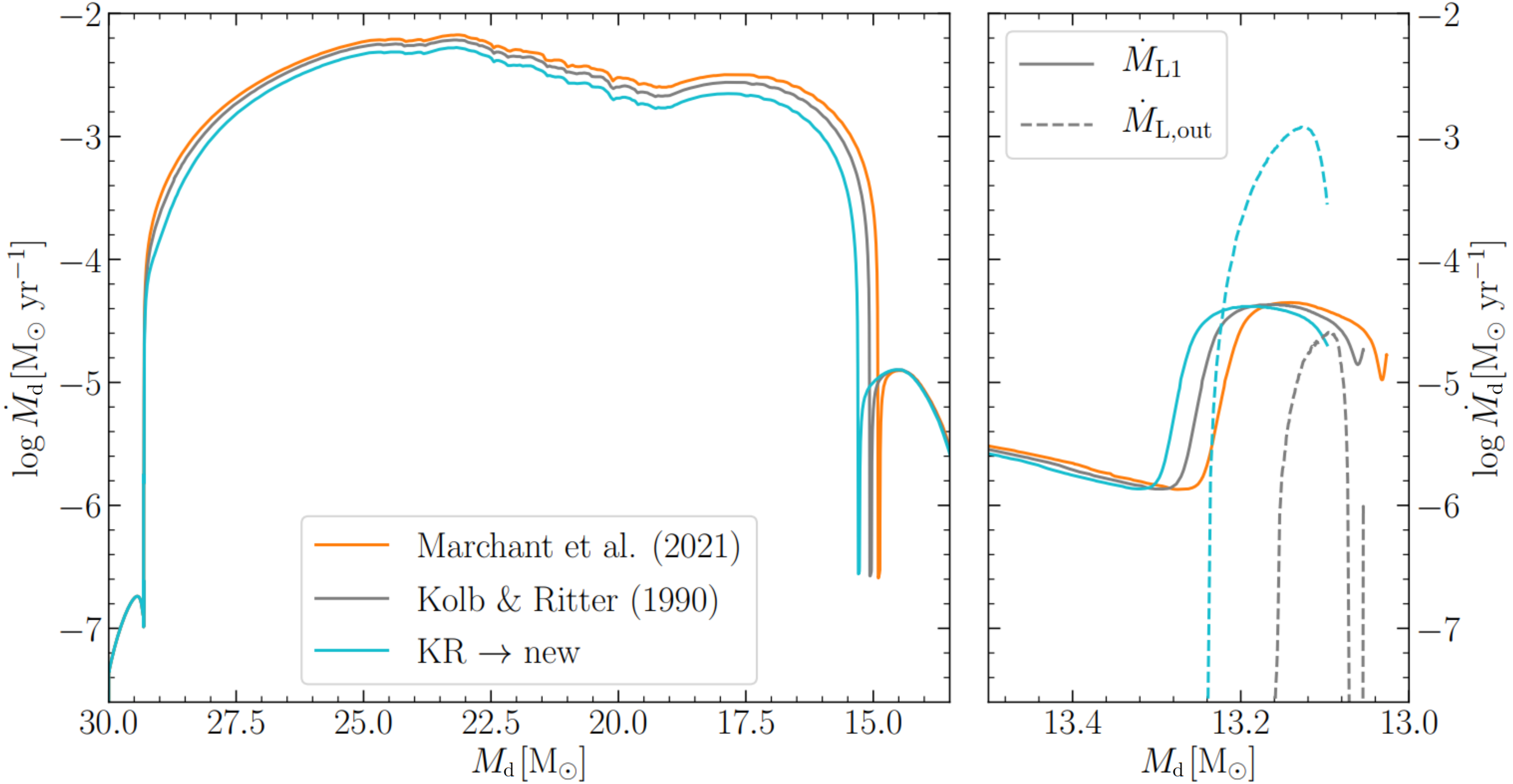


RESULTS



$\dot{M}_{\rm M}$
$\dot{M}_{\rm KR}$
$\dot{M}_{\rm new}$ $(\Delta N_{H_P} = 0.5)$
$\dot{M}_{\rm new}$ $(\Delta N_{H_P} = 1.0)$
$\dot{M}_{\rm d}/\dot{M}_{\rm KR}$
$\Delta R_{\rm d}/H_{P,\rm ph}$
$M_{\rm d}/{\rm M}_\odot$
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
20
10
0
30.0
27.5
25.0
22.5
20.0
17.5
15.0
12.5

RESULTS



−2
−3
−4
−5
−6
−7
$\log \dot{M}_{\rm d}\,[{\rm M_\odot\ yr^{-1}}]$
30.0
27.5
25.0
22.5
20.0
17.5
15.0
$M_{\rm d}\,[{\rm M_\odot}]$
Marchant et al. (2021)
Kolb & Ritter (1990)
KR → new
$\dot{M}_{\rm L1}$
$\dot{M}_{\rm L,out}$
13.4
13.2
13.0

CURRENT WORK IN EQUATIONS

START

- radiation hydrodynamics equations in the flux-limited diffusion approximation in the mixed-frame formulation (e.g. Calderón et al. 2021):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v}) + \nabla P_{\text{gas}} + \lambda \nabla E_{\text{rad}} = -\rho \nabla \phi_{\text{R}},$$

$$\frac{\partial (\rho \epsilon^*)}{\partial t} + \nabla \cdot (\rho \epsilon^* \boldsymbol{v} + P_{\text{gas}} \boldsymbol{v}) + \lambda \boldsymbol{v} \cdot \nabla E_{\text{rad}} = -c \rho \kappa_{\text{P}} \left(a T^4 - E_{\text{rad}}^{(0)} \right),$$

$$\frac{\partial E_{\text{rad}}}{\partial t} + \nabla \cdot \left(\frac{3-f}{2} E_{\text{rad}} \boldsymbol{v} \right) - \lambda \boldsymbol{v} \cdot \nabla E_{\text{rad}} = c \rho \kappa_{\text{P}} \left(a T^4 - E_{\text{rad}}^{(0)} \right) + \nabla \cdot \left(\frac{c \lambda}{\rho \kappa_{\text{R}}} \nabla E_{\text{rad}} \right),$$

$$\boldsymbol{F}_{\text{rad}}^{(0)} = -\frac{c \lambda}{\rho \kappa_{\text{R}}} \nabla E_{\text{rad}}^{(0)},$$

$$P_{\text{rad}}^{(0)} = f^{(0)} E_{\text{rad}}^{(0)},$$

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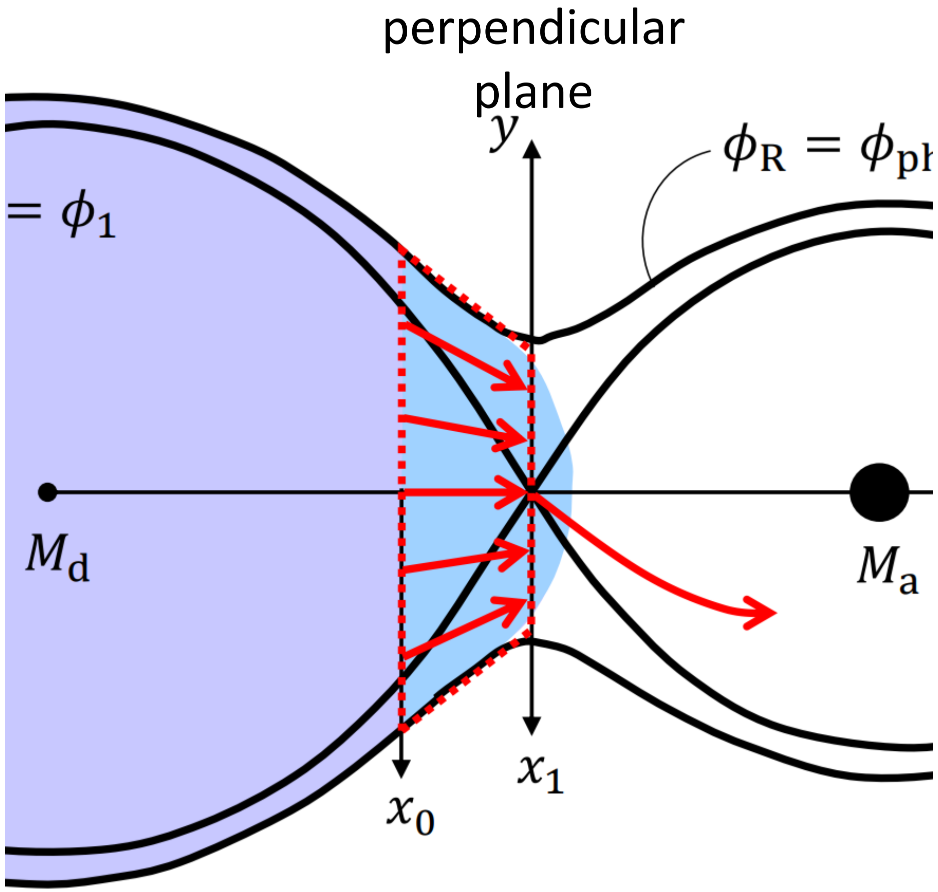

ASSUMPTIONS:

1. Stationarity
2. Gas flow – 1D
3. LTE: $aT^4 - E_{\rm rad} = 0$
4. Optically thick limit: $\lambda \rightarrow 1/3$

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END

- 1D radiation hydrodynamics equations with the Roche potential and radiative flux:

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x} = 0,$$
$$v\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}P_{\rm gas}}{\mathrm{d}x} = \frac{\kappa}{c}F_{\rm rad} - \frac{\mathrm{d}\phi_{\rm R}}{\mathrm{d}x},$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[(\epsilon_{\rm tot}\rho + P)\, v + F_{\rm rad}\right] = 0,$$
$$F_{\rm rad} = -\frac{c}{\kappa}\frac{1}{\rho}\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}x},$$

where: $P_{\rm rad} = \frac{1}{3}aT^4, \quad E_{\rm rad} = aT^4.$

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mixed-frame formulation:

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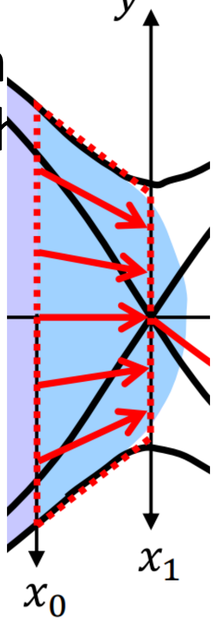

ND

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ASSUMPTIONS:

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